To cognize is to categorize: Cognition theory could be Category theory like

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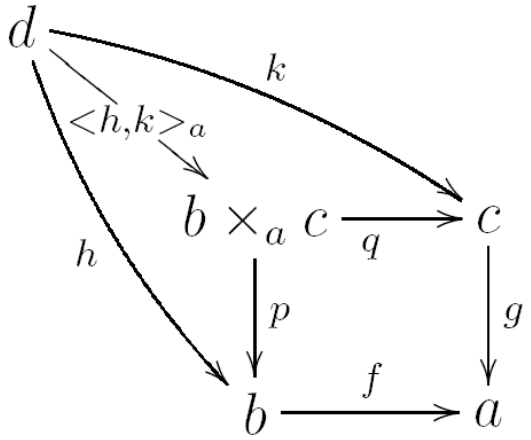

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RESEARCH INSTITUTE
UNANSWERED QUESTIONS
UNQUESTIONED ANSWERS

The challenge

Formalize …COGNITION
Agenda:
Provide a theoretical framework able to express formally knowledge indepently of any hermeneutics

What we need

THEORY

•Mathematics to remove ambiguity in the descriptions
•Semantic model that provides an analytical framework for understanding the capability of the system to acquire cognitive, autonomic, emotional aspects

Thats all?

The shift towards structure:

- The dictionary defines structure as “the arrangement of and relations between the parts or elements of something complex”.
- Structure is a word used in many different contexts and situations, for example in architecture a structure is a man-made object fixed to a surface -e.g:building; in social sciences, structure is intended as a human organisation with some common characteristic or goal and in molecular biology, the structure of a cell is its shape or geometric appareance
- Mathematics aims to give the formal descriptions that convey the general structure. It becomes relevant to science because is able to uncover the structure of the phenomenon studied by the social or natural scientist, capturing the patterns and regularities that are hold by the system under study.

Structure Form, the melancolic loop

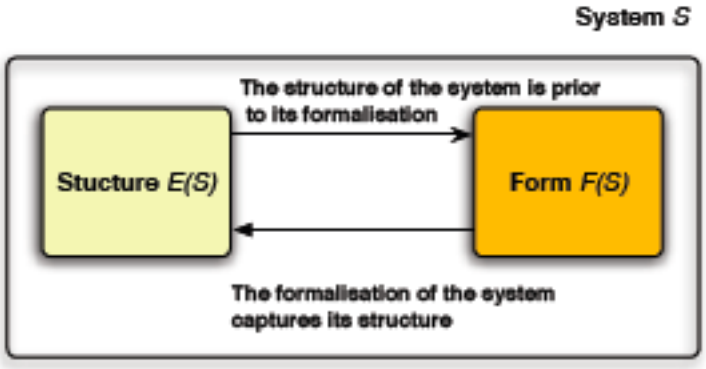


Figure 1.8: The structure is prior to its formal and the formalisation captures the structure of the system

The idea is to go beyond the predominant view of structure as a geometrical shape (life sciences) or as a construction permanently fixed to Earth's surface (architecture) and translate the mathematical concept of structure into the arena of the natural sciences, in particular in the cognitive science

Formal

- Descartes' invention of analytical geometry supposes an astounding conceptual tool
- Dont forget that was a Violeant Irruption of numbers into geometry
- This paved the way, three centuries later! with the work of Lobachenski and Bolyai, of non euclidean geometry and its application in quantum physics
- Formal systems, expresed by diagrams, equations or whatsoever can be an extraordinarily efficient and successful mechanisms to either explain and predict any phenomena, or to represent any feature of the world.
- The point to be retained is that the formalisations are methodological tools and not ontological simplifications.

Formal

- *"and as honest scientists, as far as we disentangle the stucture we need to begin to describe it in formal terms."* [Luce]
- *"Scholars in social sciences seem to be comfortable with the belief that biology and natural selection successfully delivered homo sapiens into the upper paleolithic and then was abandoned to the ministrations of culture."[*Smolin]
- *"power to operate with temporary emotional symbols and to organize out of them a semipermanent, recallable language. If one is not able to do this, one is likely to find that his ideas evaporate from the sheer difficulty of preserving them in an as yet unformulated shape."* [Wiener]

Obstacles to formalisation:

- The definitions in the dictionary are circular

  -The indecomposable elements are mathematical entities used in topology that can be seen as a elementary particle with a minimal amount of ambiguity in their definition.
  -The indecomposable elements holds the Indecomposability principle, established by Brouwer that affirms that the continuum cannot be partitioned into two nonempty pieces.

- The plurality of definitions

  -In mathematics NO always an unique definition for every mathematical concepts not even for those as important as function or structure.
  -This is because these are concepts used in several areas of mathematics, in many different ways, so no single definition of function has been universally adopted.
  -This lack of unity in the definition even for some of the most fundamental concepts, ought not to drive us to wrong conclusions about the useless or impossibility of the formal program.
  -For example, in set theory, morphisms are functions; in linear algebra, linear transformations; in group theory, group homomorphisms; in topology, continuous functions

Obstacles to formalisation:

- Formalisation as intuitive instantiation of Platonic forms

  -Gödel himself is a champion of the platonism in mathematics, not in geometry which He conceives as physically grounded in the world but with set theory.
  -The objects of set theory, are Platonic Forms because are non-physical yet objective. Due to the intuition faculty is possible to perceive truths about these objects with which we have no physical interaction.

Let us for a moment assume the Gödel's platonism credo for set theory; then the Platonic Forms are committed to some law or order and as are known through intuition, it follows that those intuitions must be governed by rules as well.
Said otherly, both intuition and set-theoretic objects, are mutually constrained by the morphological laws

- This is a dangerous common place that mus be debunked:
  - Primo, even in the case that formalisation was une *chôse pour des elites*, this elite should be listened to, at least as much as those lacking the formalist skills.
  - Secondo, there is no such a thing as unstructured cognitive state, any mental content to be attaint, needs of the intermediation of perceptual schemas (forms) that configures the agent's perception

Category Theory

•The mathematical semantic model is based upon CATEGORY THEORY

•CT is the mathematical theory of structure.

•CT express relationships between structures

•In studying brain structure we can't prescind of use the mathematics of the structure

Category Theory

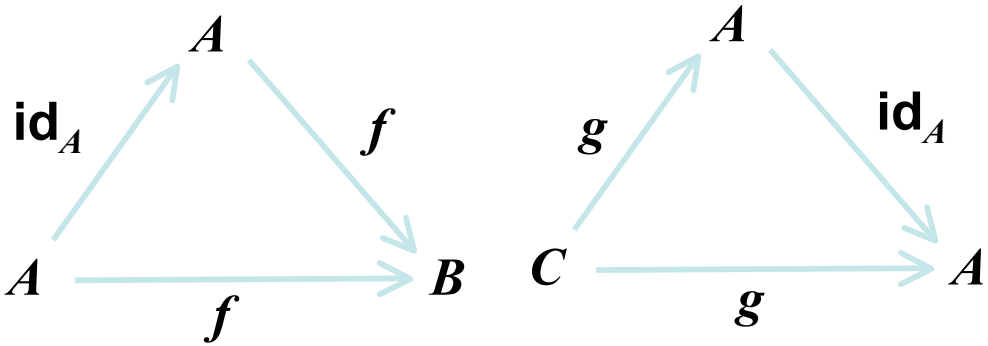
1940s Eilenberg Mc Lane

A category is composed of ONLY!

- Objects A, B, C …
- Morphisms or arrows f:a $\rightarrow$ b
  - dom(f)=A cod(f)=B
- The identity arrow I_a identifies an object a I_a :a->a
- Arrows are composable if the source of one forms the target of the other:dom(g)=cod(f)
- f:a $\rightarrow$ b, g:b $\rightarrow$ c has composite h:a $\rightarrow$ c
- Composition is associative:
  - f: a $\rightarrow$ b, g: b $\rightarrow$ c, h: c $\rightarrow$ d
  - f o (g o (h)) = ((f) o g) o h

Category Theory

Every object has an identity morphism



$f \circ \mathrm{id}_A = f$

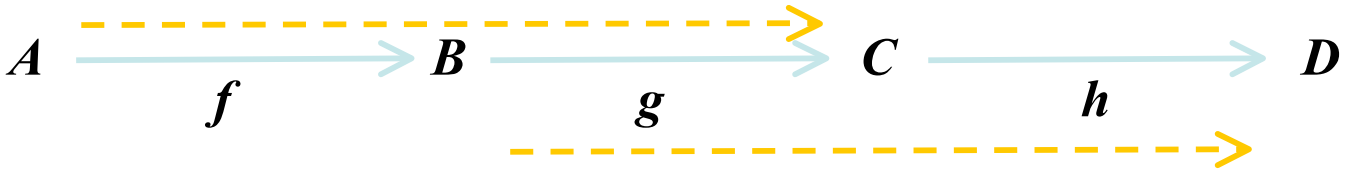
$\mathrm{id}_A \circ g = g$

A diagram expresses a system of mathematical constraints associatiated with a category

The diagram of the figure conmutes .
A conmutative diagram is assotiated with a system of equations involving composition of morphisms. equations

Composition is associative

$$h \circ (g \circ f) = (h \circ g) \circ f$$



Category Theory

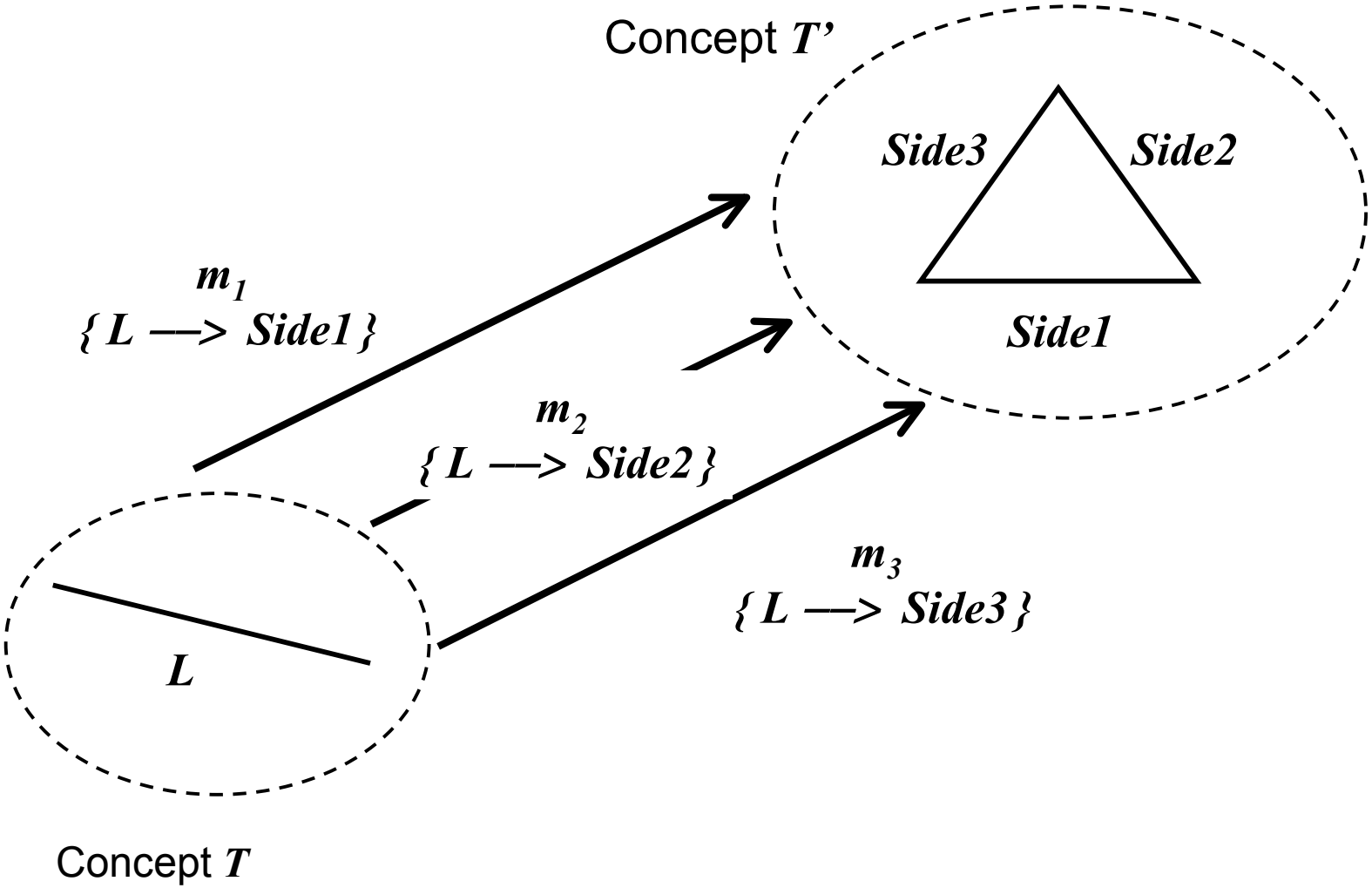
Concept morphism

m: X —> Y

The morphism m: X —> Y specifies the mapping of the Concept X into the Concept Y.

This in effect describes the role that the objects described by X (circles here) play within Y (rectangles containing circles).

Multiple morphisms



Category Theory

0

empty category -- no objects, no arrows

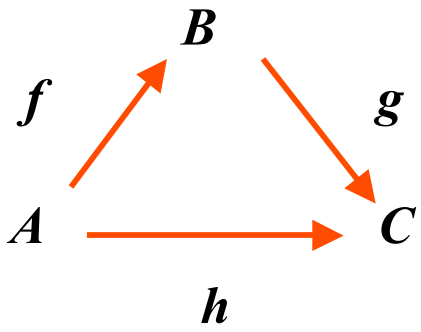
1

(discrete) category with one object and one identity arrow
e.g. A, 1_a: a->a

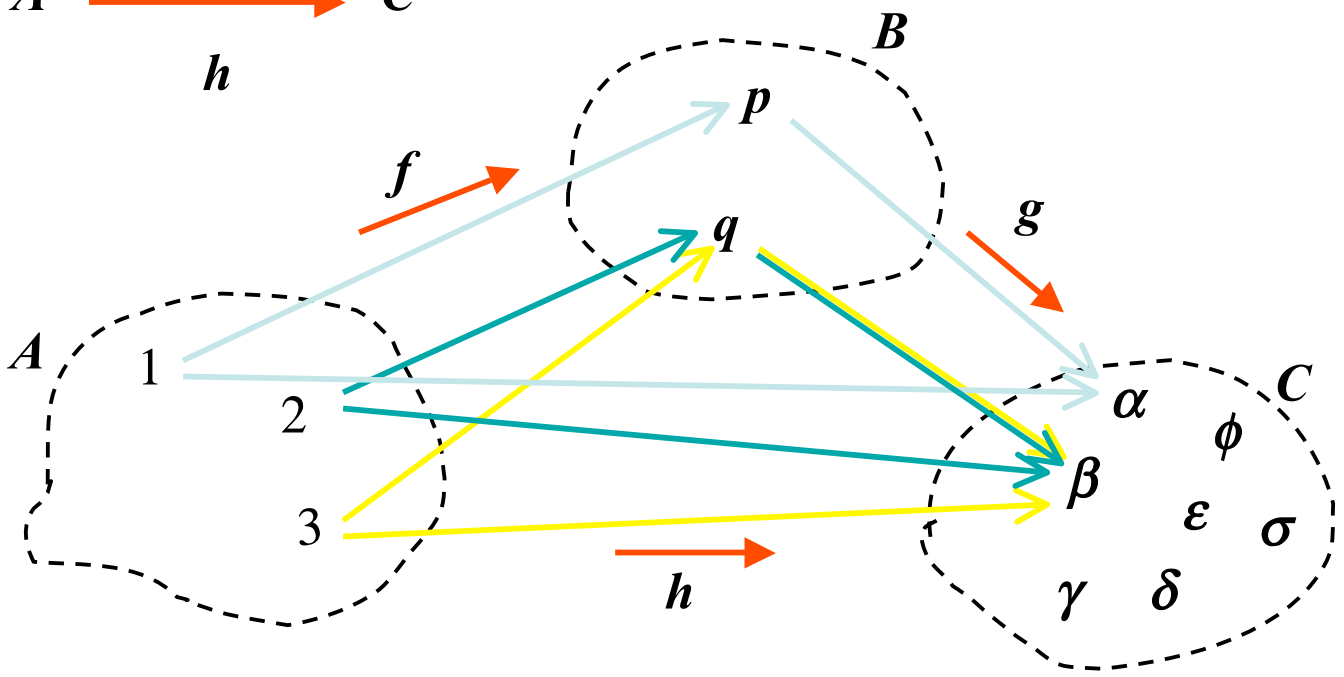
Set

The objects are sets
The arrows are total functions between the sets
e.g. Sets X, Y, Z
Total functions a:X-> Y, b:Y->Z

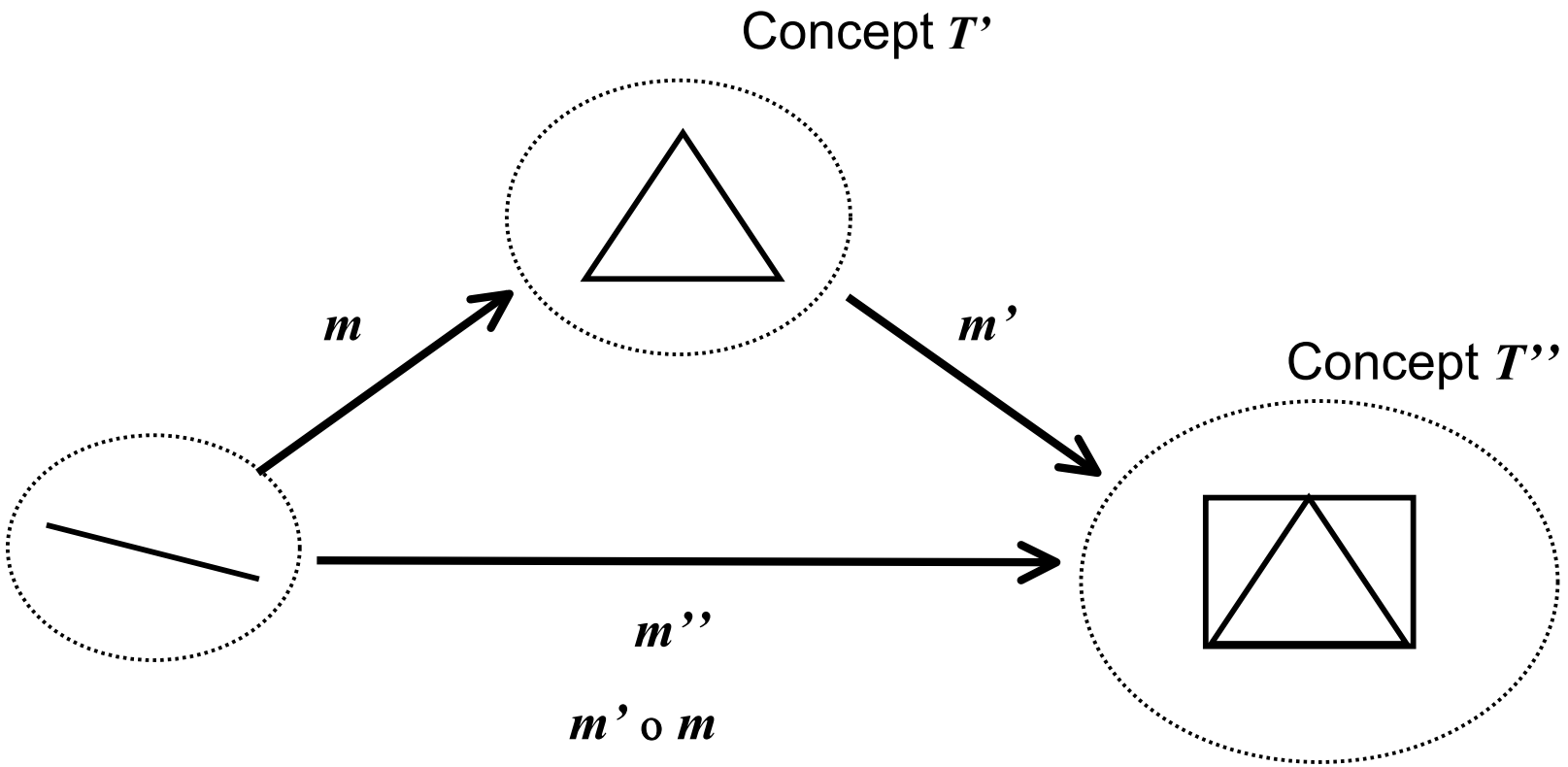
Mapping of objects and morphisms

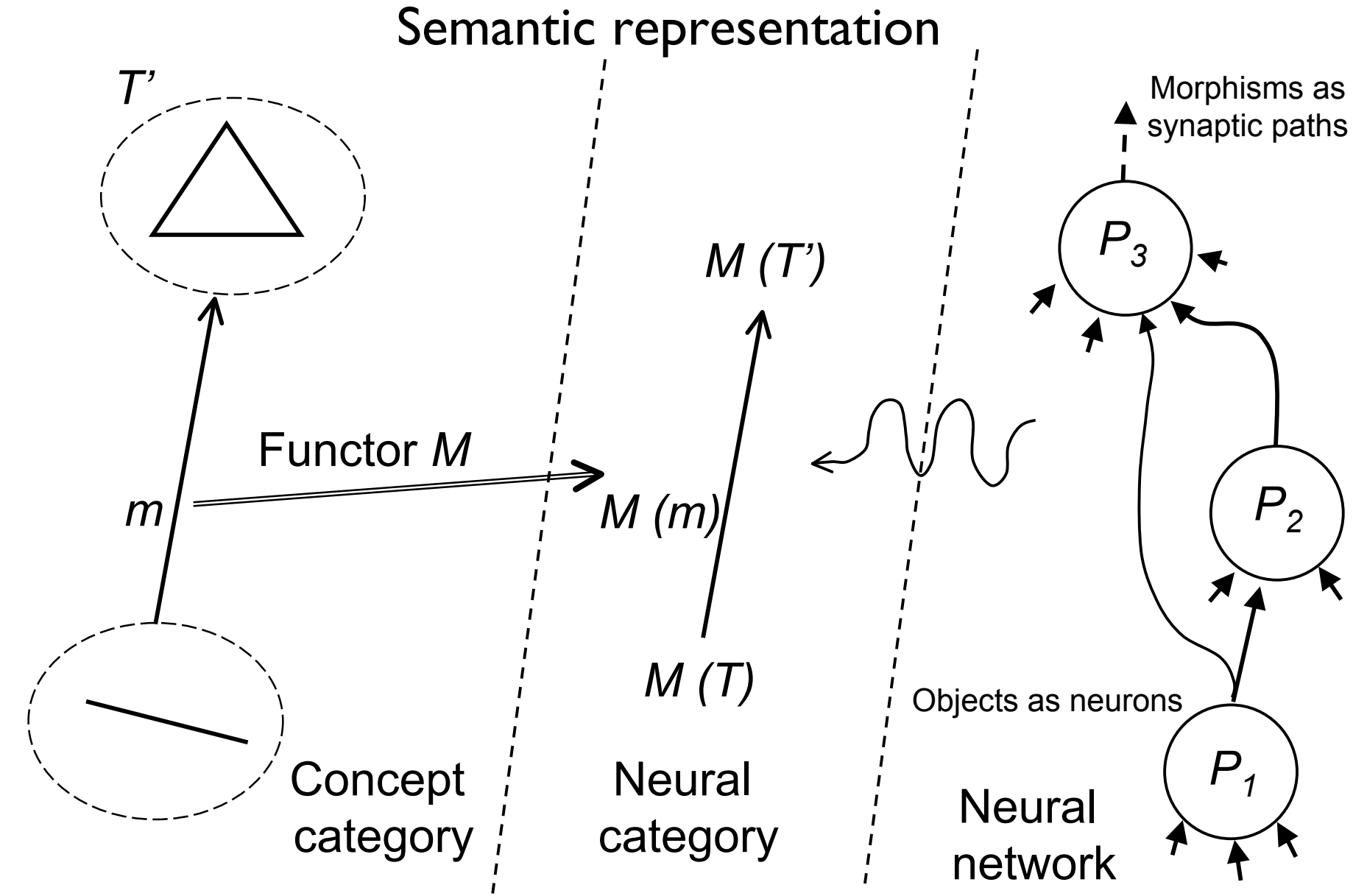


$$h = g \circ f$$
$$h(x) = g(f(x))$$



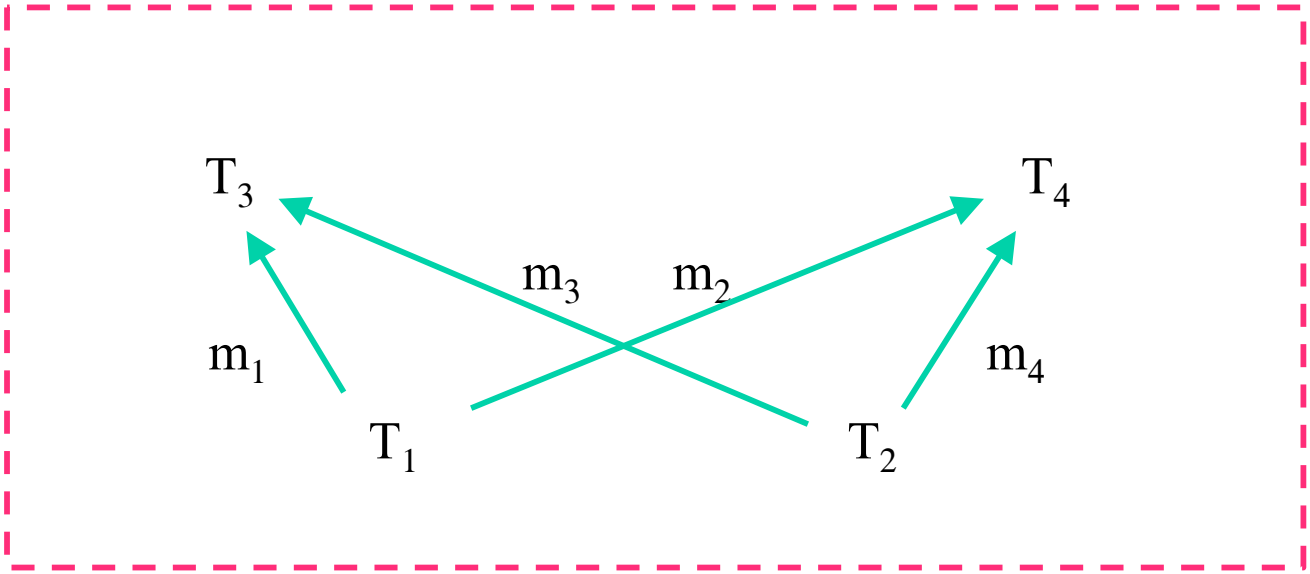
A category of concepts
and concept morphisms




Semantic representation
T'
Morphisms as synaptic paths
M (T')
P3
Functor M
m
M (m)
P2
M (T)
Objects as neurons
P1
Concept category
Neural category
Neural network

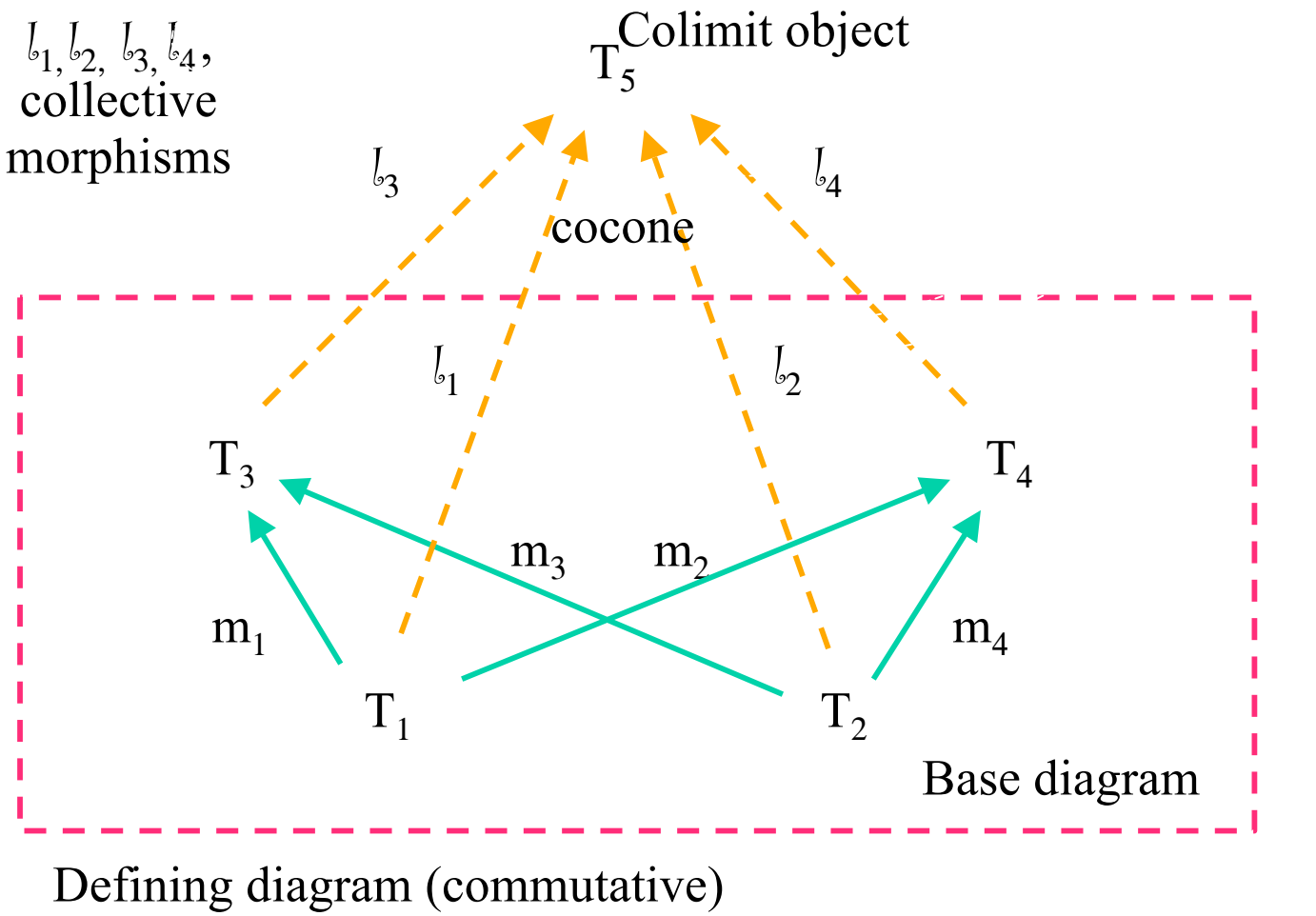
Category Theory

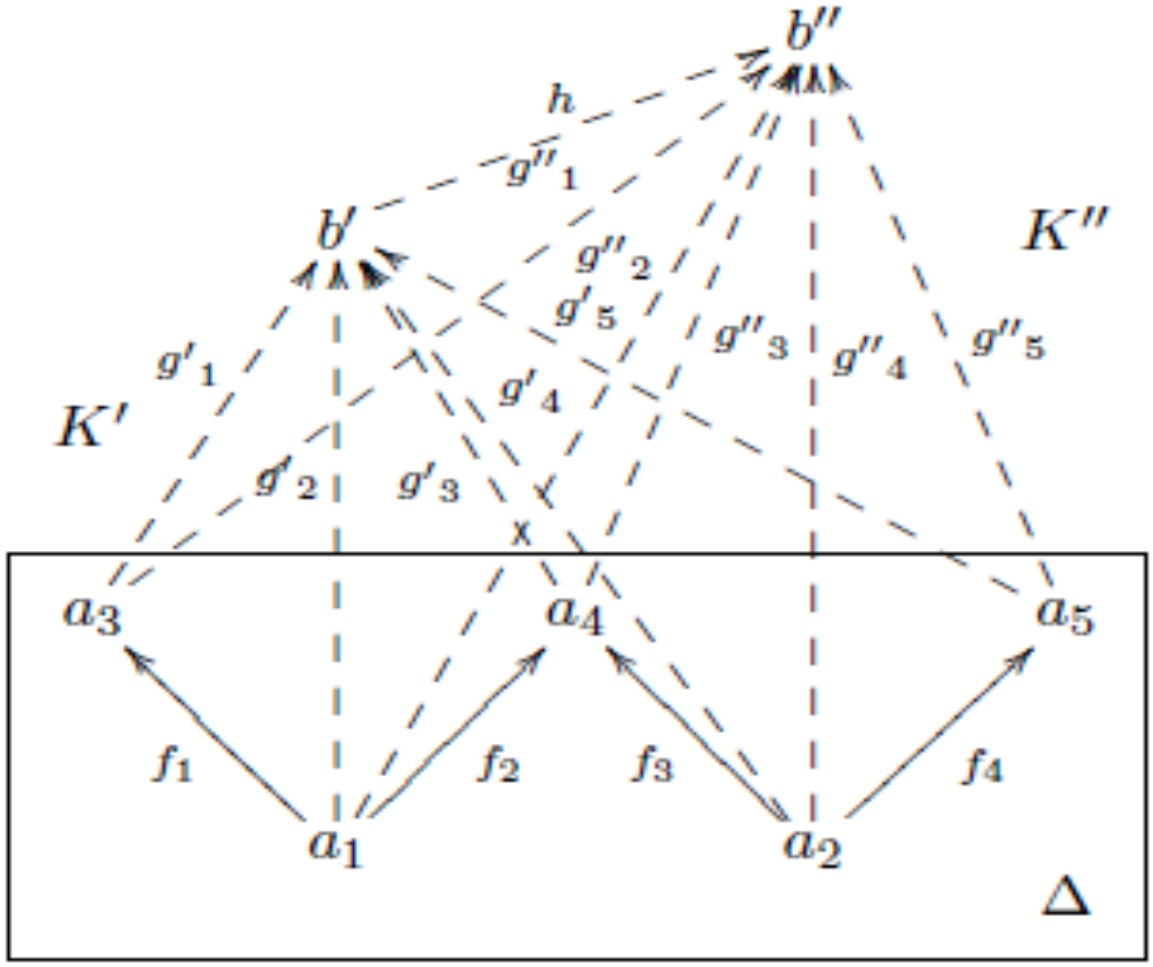
A diagram: objects and morphisms form a category



Colimit

Colimit SPECIALISATION

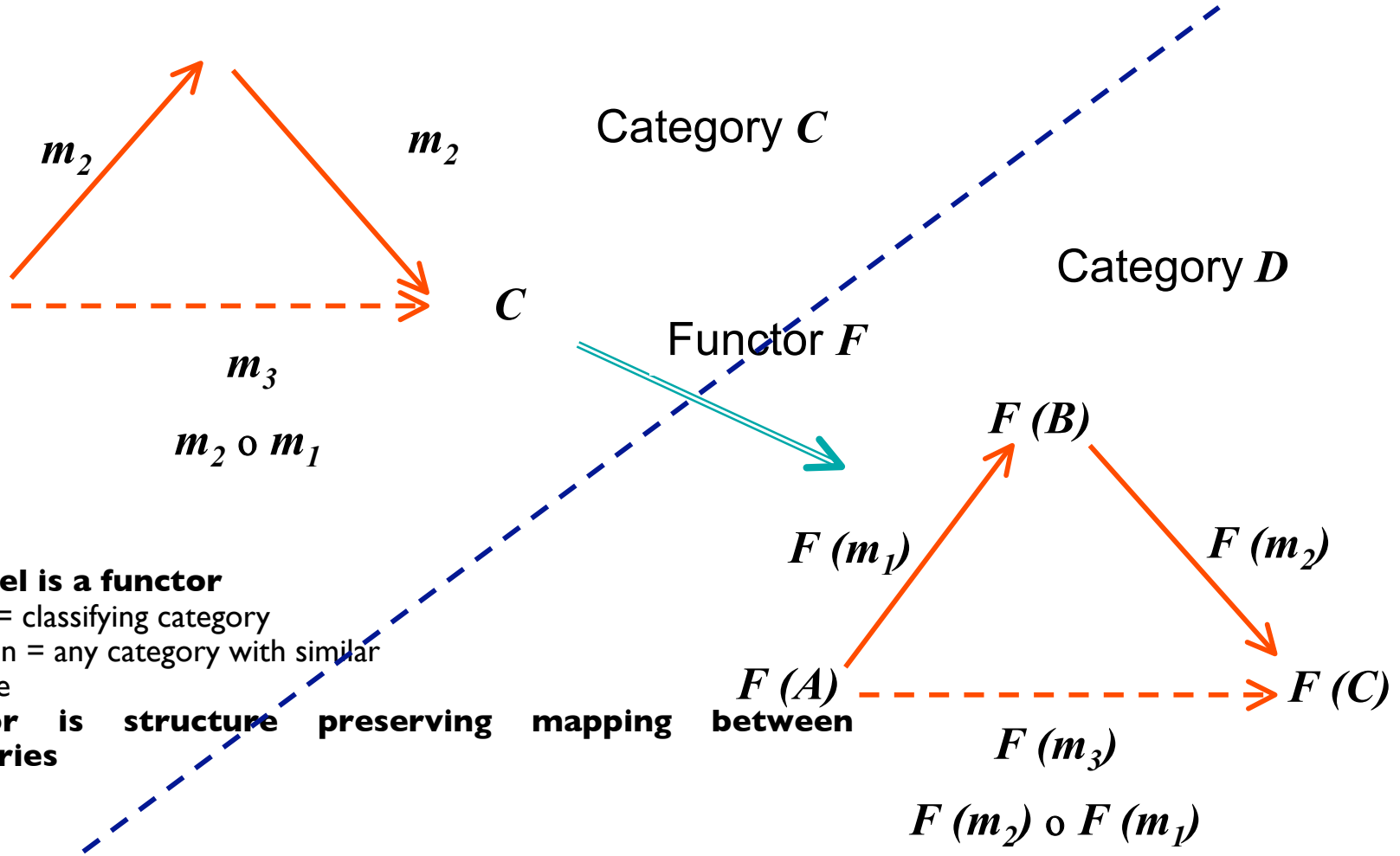




A cocone morphism $h: K' \longrightarrow K''$ in $\mathbf{coc}_\Delta$

Colimit: Least specialization

Functor



A model is a functor
domain = classifying category
codomain = any category with similar structure
Functor is structure preserving mapping between categories

Relevant Categories

•The Neural system Consists of neurons(intermediate, effector, receptor..) and synapses between them, its dynamics results from the propagation of an action potential from a neuron to other neurons through synapses

•We model the configuration of the neural system at *t* by a graph: its vertices model the states N(*t*) of the neurons N existing at *t*, an arrow from N(*t*) to N(*t*'), models the state at *t* of a synaptic path *s* from N to N'; it is determined by the strength $w_s(t)$

Equipped with the composition defined by concatenation, this graph becomes the ***category* Neur$_t$** *of neurons* at *t*

Category of Neurons

Over time, say from t to a later time t', some neurons are lost, while a few new neurons can grow, the number or wights of synapses can vary.

This change of configuration is modeled by a partial map from Neurt -> Neurt' sending the state of a neuron N at t to its state at t' if N still exists, and similarly for a link.

This transition defines a functor from a sub-category of Neurt to Neurt'

Relevant Categories

Mental Images The mental concept for a new item O is generated through the activation of a specific pattern P, thus forming a short-term memory of O.

By Hebb rule, their coordinated activation at *t* increases the strength of the distinguished links between them and the same repeats at each successive perception of O. Over time, the pattern P will take its own identity, being able to act as a synchronous assembly of neurons.

However we cannot identify the pattern as such with the mental image of O until we hava a semantic model P != Mental Concept O (this is specially true for complex O)

Let P (p_i, ,Ω) Ω is the range of output values of node p_i of the pattern P

For Place cell the concept (spatil location) can be modeled by a single Neuron

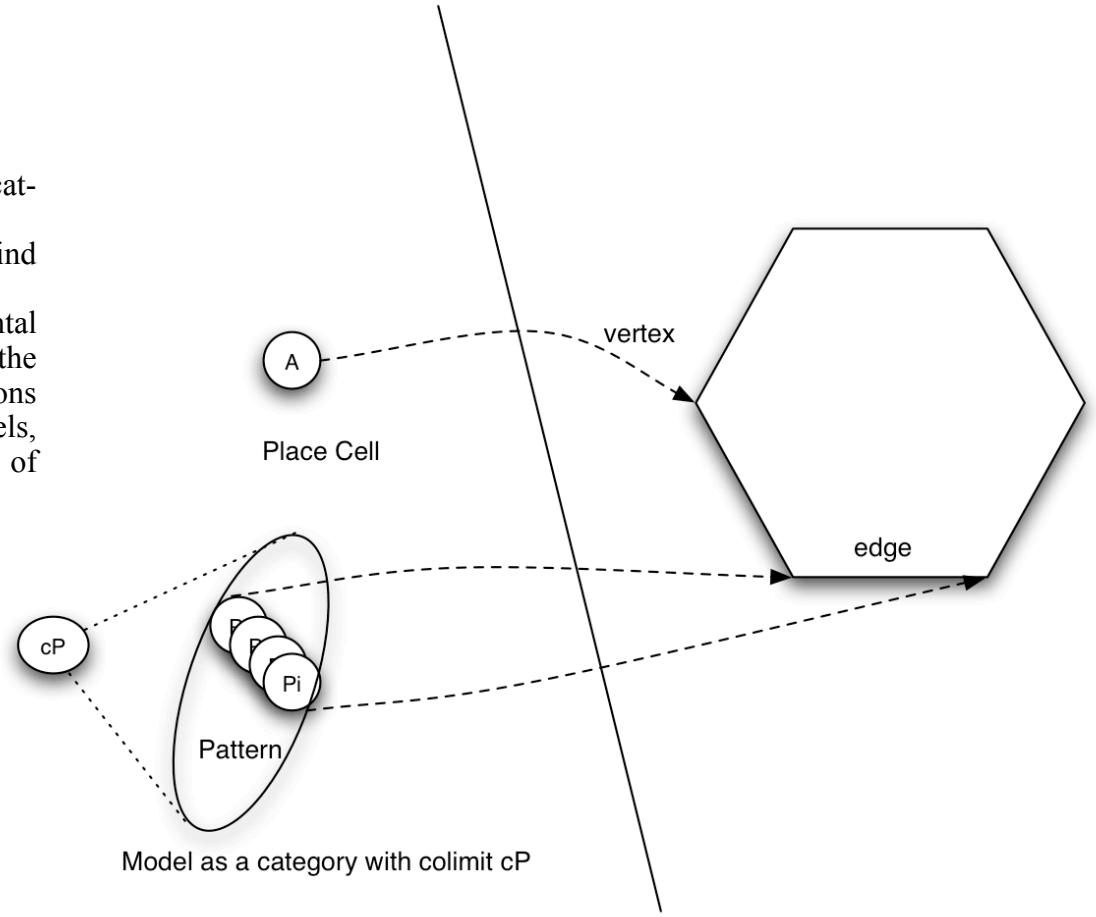
For more compelx concepts and emotions we need to model with cat-neuron

Depending on the context, the same sobject O can activate more or less different patterns acting as synchronous assemblies of neurons, and these patterns are not necessarily interconnected . **"*degeneracy* of the neuronal encoding"** (Edelman, 1989)

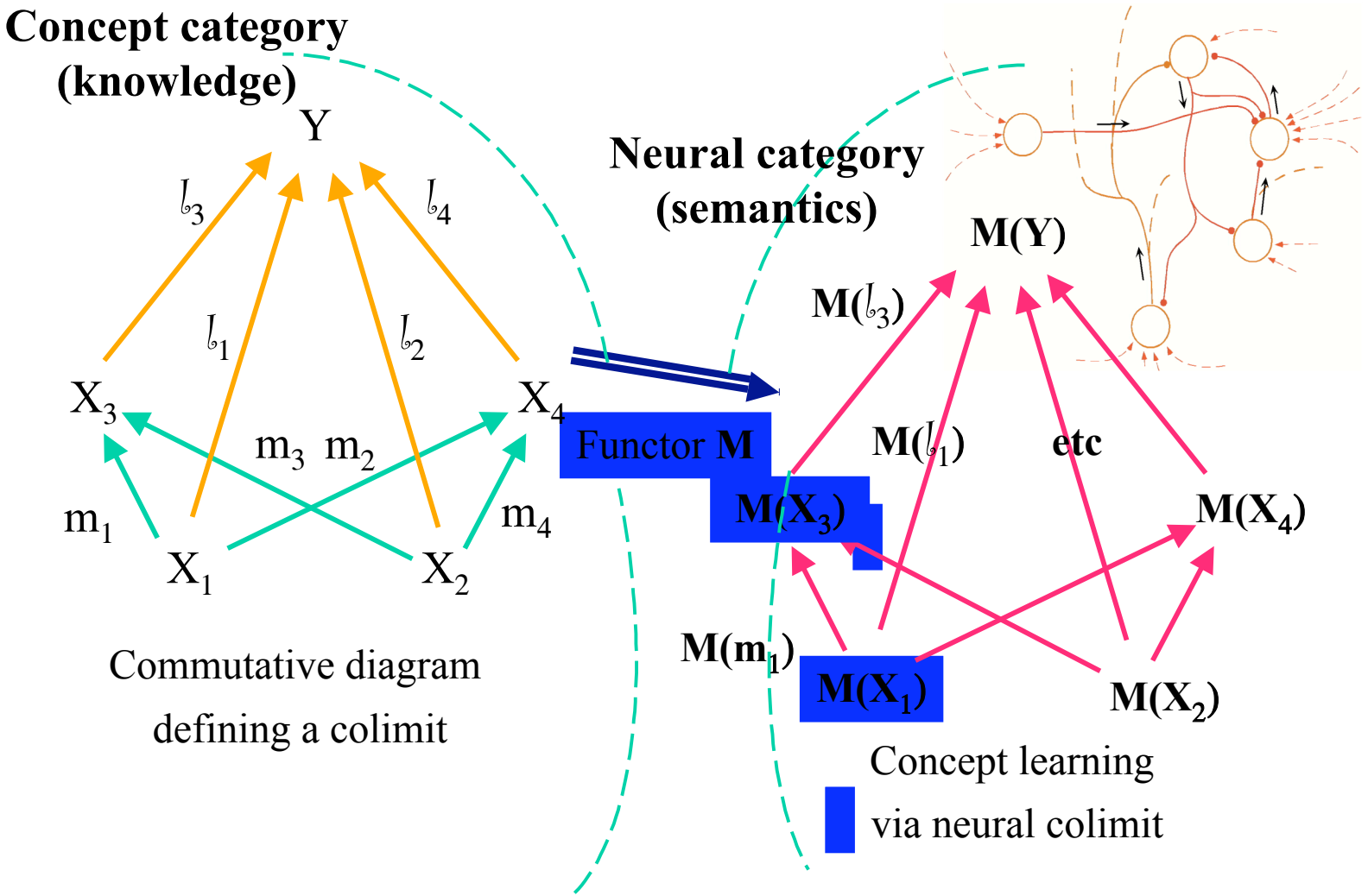
Algebra of concepts

The animal has a hierarchy of mental objects, from the mental image of a simple object modeled by a single neuron, to mental objects activated by a synchronous assembly of neurons, up to more complex mental objects combining more elementary ones

The neurons identified to cat-neurons of level 0.
The cat-neurons of level 1 bind patterns of neurons.
To model more complex mental objects, we have to iterate the construction and form cat-neurons of increasing complexity levels, binding patterns of cat-neurons of lower complexity

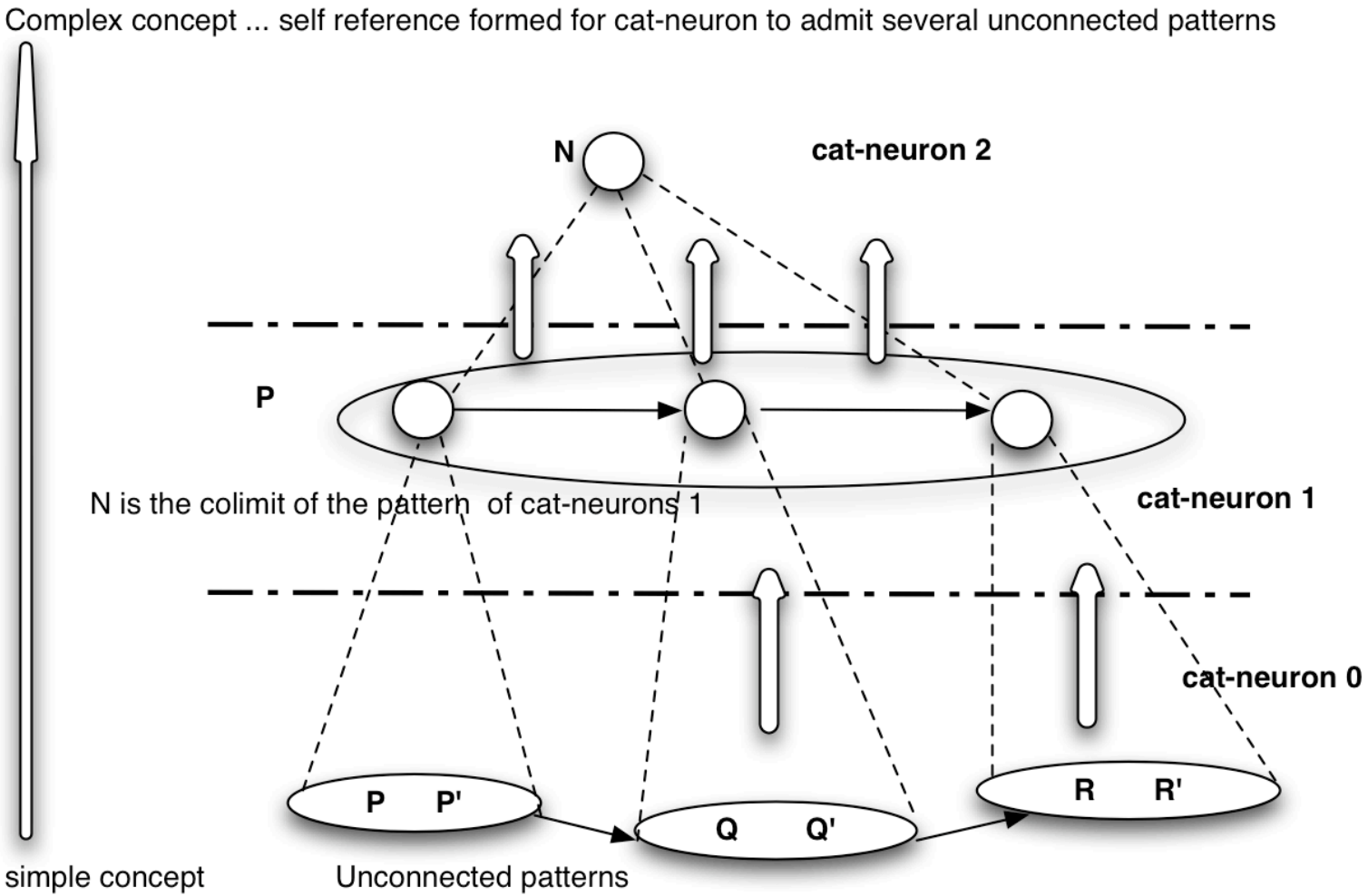


Functors Express Semantic (Structural) Mappings



Algebra of concepts

•The neurons cat-neurons of L0
•The cat-neurons of L1 bind patterns of neurons , are colimits of cat-neurons of L0
•To model more complex mental objects, we need to iterate and form cat-neurons of increasing complexity levels, binding patterns of cat-neurons of lower complexity.
•The construction of cat-neurons interpreted as developing an algebra of mental objects
•A cat-neuron ***N*** of level *n* "emerges" in a complexification process to bind a pattern of cat-neurons of strictly lower levels
•Emergence mathematically defined as the binding of several non interconnected patterns of lower cat-neurons
• In CT the binding is a colimit = mathematical explicit description
•The root of the emergence of higher cognitive processes, up to consciousness is the degeneracy of the neuronal coding
•A mental object modeled by a cat-neuron "supervenes" on physical brain processes via the stepwise construction from the neuron level up assemblies of asynchronous neurons



The iterative building of cat-neurons binding patterns of cat-neurons of lower levels, models the computation on mental objects, to form more complex ones. It give explanatory power for the construction of an algebra of mental objects

Some literature